Differential Equations (D.E)

In Mathematics diff. eqn is an eqn that relates one more functions and their derivatives.

$y + \frac{dy}{dx} = 5x$. → equation is an

$\frac{dy}{dx}$ → derivative.

eg: of diff. eqn with the function y and its derivative $\frac{dy}{dx}$.

Geometrical defn of ~~derivate~~ diff. eqn. —

Diff. eqn geometrically represents either a family or families of curves or some individual curves.

Direction fields

Direction field is defined as the collection of small line segments passing through various points having a slope that can satisfy the given diff. eqn.

Order and degree of a D.E

A diff. eqn involving a single independent variable and hence only ordinary derivatives is called ordinary diff. eqn (O.D.E).

A diff. eqn involving more than one independent variable and hence partial derivatives is called partial diff. eqn (P.D.E)

The order of a D.E is the order of highest derivative occuring in it.

The degree of D.E is the degree of the highest derivative occurs in it.

eg:- 1) $\frac{dy}{dx} = \tan x$.

order – 1, degree – 1

2) $y'' + 9y = 0$.

order – 2, degree – 1

3) $y\left(\frac{dy}{dx}\right)^2 + 2x - y = 0$

order – 1, degree – 2.

Solution of First order ordinary D. E.

A function $y = g(x)$ is called a solution of a given first order D. E on some interval, say $a < x < b$ if $g(x)$ is defined and differentiable throughout that interval and is such that the eqn becomes an identity when y and y' are replaced by g and g' respectively.

Eg :- The function $y = g(x) = e^{2x}$

is a soln of the first order D. E

$\frac{dy}{dx} = 2y$ for all x.

i.e $g(x) = e^{2x}$

$g'(x) = 2e^{2x}$

Substituting in $\frac{dy}{dx} = 2y$

$\Rightarrow g'(x) = 2g(x)$

$2 \cdot e^{2x} = 2 \cdot e^{2x}$

$\therefore g(x)$ is a soln.

General Soln and Particular soln.

The soln of a first order ordinary D.E which contain one arbitrary constant (say c, which can take infinitely many values) is called a general soln.

A soln obtainable from the general soln by giving particular value to the arbitrary constant c is called a particular soln.

The geometrical representation of the general soln is an infinite family of curves is called integral curves.

eg:- $y = \sin x + c$, where c as arbitrary is a general soln of the D.E $\frac{dy}{dx} = \cos x$.

Each of the soln's $y = \sin x$

$y = \sin x + 3$, $y = \sin x - 4$ is a particular solution of the D.E.

Question

verify that the given function is a soln of the given diff. eqn

$y = c e^{-8x}$, $y' + 8y = 0$.

$y' = c \cdot e^{-8x} \cdot -8$

$= -8c\, e^{-8x}$

then $y' + 8y = -8c\, e^{-8x} + 8 \cdot c\, e^{-8x}$

$= 0$

verified.

First order ordinary diff. eqns.

Separable eqns.

Consider the diff. eqns of the form

$g(y) \frac{dy}{dx} = f(x)$ —— ①

(separable eqns).

Then we integrate on both sides w.r.t to x, obtaining

$$\int g(y)\, dy = \int f(x)\, dx + c.$$

where c is arbitrary constant.

This method is called method of separating variables or variable separable method.

eg:-1) Solve the diff eqn

$10\,y\,y' + 3x = 0$.

The given diff eqn can be written as

$10\,y\,\frac{dy}{dx} + 3x = 0$.

$10 \cdot y\,\frac{dy}{dx} = -3x$.

$10 \cdot y \cdot dy = -3x\,dx$

which is in separable form

Integrating both sides we get.

$\int 10 \cdot y \cdot dy = \int -3x\,dx$

$10 \cdot \frac{y^2}{2} = -3\,\frac{x^2}{2} + C$.

$5y^2 = -\frac{3}{2}x^2 + C$.

==.

2) Solve $\frac{dy}{dx} + 2xy = 0$.

By separating variables

$\frac{dy}{dx} = -2xy$

$\frac{dy}{y} = -2x\,dx$

$\int \frac{dy}{y} = \int -2x \cdot dx$

$\log|y| = -2\,\frac{x^2}{2} + C$.

$\log|y| = x^2 + C$ //

Questions

Solve the following diff eqns.

1. $y' = ky$.
2. $y' - 2y + a = 0$.
3. $xy' + by = 0$.
4. $x \log x \cdot y' = y$.

Exact D. E.

A D. E is said to be exact if it can be derived from its primitive (solution) directly by diff. with out any subsequent multiplication, elimination etc.

Thus the D. E

$M\,dx + N\,dy = 0$, where both M and N are functions of x and y is exact if there exist a function u of x and y such that

$M\,dx + N\,dy = du$

eg:- Find the exact diff. eqn that has $x^2 + y^2 = c$ as the general soln.

Soln Corresponding to the given